Exercise 7

For each of the following integral equations, classify as Fredholm, Volterra, or Volterra-Fredholm integral equation and find its kind. Classify the equation as singular or not.

$$\frac{1}{6}x^3 = \int_0^x (x - t)u(t)\,dt$$

Solution

This is a Volterra integral equation because one of the limits of integration is not constant. It is of the first kind because the unknown function u appears only inside the integral. It's inhomogeneous because of the $(1/6)x^3$ on the left side. It's not singular since neither of the limits of integration are infinite and the integrand does not become infinite in the interval of integration.